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The Relationship of Cognitive Attributes of Mathematics and Science

Kim, Sun Hee
(Assistant Professor, Silla University)*
Kim, Soojin
(Research Fellow, Korea Institute for Curriculum and Evaluation)**
Kim, Ji Young
(Teacher, Myoungil Middle School)

≪ ABSTRACT ≫

The purpose of this study is to diagnose students' cognitive attributes in mathematics and science based on the Fusion model, one of several Cognitive diagnostic models. The Cognitive diagnostic model measures several attributes from a single item and analyzes the results based on the fields of education evaluation and measurement theory. The test, composed of multiple-choice problems in mathematics and science, was administered to the eighth graders in a middle school located in Seoul. Using this data, this study examined the correlation of cognitive attributes for mathematics and science and attempted to identify the attributes that influence achievements in mathematics and science through regression analysis. The results suggest a significant correlation of cognitive attributes for both mathematics and science, as well as several abilities in mathematics that can help in solving scientific problems.

Key Words : mathematics, science, cognitive attributes, cognitive diagnosis theory, achievement

* 제1저자
** 교신저자, sjkim@kice.re.kr

Ⅰ. Introduction

Which ability does a student need when solving problems in mathematics and science? How can this ability be measured? The recent educational reforms in the mathematics and science fields require more advanced thinking and inquiry abilities of students and the revision of Korean national curriculum has been done with this educational reform trends. Especially, Ministry of Education, Science and Technology in Korea emphasizes the STEAM(science, technology, engineering, art and mathematics) that is interdisciplinary education.

Mathematics education emphasizes the importance of figuring out the concept, analyzing and solving the problem, communicating and inferring the data, and connecting to internal or external contexts of mathematics. Science education stresses the problem-solving process through applying scientific principles and the procedure of raising and utilizing scientific process skills. Accordingly, the teaching, learning, and evaluating processes for mathematics and science should be modified to include an evaluation of problem-solving skills.

Mathematics has been valued and justified in education as the foundation of other academic fields. In particular, scientific facts are becoming theories based on mathematics. A mathematical representation can generalize observed results. Such a mathematical representation can also simplify and organize a scientific concept. When mathematics is incorporated into a science lesson, the two disciplines can complement each other in such a way that the learning of both is enhanced. The scientific processes of observation and data collection are incomplete without the use of mathematics to analyze data and quantitatively reveal relationships in order to draw conclusions.

Basson (2002) asserted that scientific concepts have their foundation in mathematical concepts. He said that a connectedness and an organization of different mathematical and scientific concepts should be enhanced and utilized. Wilhelm and Walters (2006) also found this to be true in their research regarding the integration of mathematics and science using an inquiry-based classroom setting. Reiss and Ruthven (2011) proposed that both science and mathematics education be encouraged, particularly where they identify significant commonalities, contrasts, or interactions between these areas. Sherrod, Dwyer and Narayan (2009) reported the development and refinement of integrated activities for middle school students taking science and mathematics. The activities gave students a chance to be scientists by investigating scientific phenomena and to be mathematicians in calculating and analyzing the data while performing these activities in an environment that supported them in constructing a more comprehensive understanding of mathematics and science. This nurturance also empowered them to make use of their skills in the real world. Goldberg and Wagreich (1989) developed an integrated program for grades one to eight. The results from their study indicated that the program had a marked improvement on the mathematics-science process skills of learners.

As mentioned previously, the learning of mathematics and science is connected to each other, and the effort to educate integrally is noted. However, some of the research topics on the connection between mathematics and science have focused on either the correlation of achievement scores or on activities of the integrated education between the two subjects. Anderson et al. (2007) indicated that the correlation is very high (r=0.97) as a analysis result of the relationship between the two achievement scores of science and mathematics in Organization for Economic Co-operation and Development (OECD) countries. A correlation between mathematics achievement and science achievement scores has shown a high connectivity between the two. However, previous studies have not fully examined which ability in mathematics helps in studying science and vice versa.

This study utilizes a mathematics and science test composed of the multiple-choice items for the eighth grader of a middle school in order to diagnose the cognitive attributes of students when solving mathematics and science problems. The Cognitive diagnosis theory measures several cognitive attributes from a single item, and analyzes the results based on education evaluation and measurement theory. Previously, Kim (2009) has studied eighth grade students' cognitive attributes in geometry of mathematics. But in this study, scientific attributes are also considered.

In this study, what we intend to research are as follows:

First, this study diagnose the cognitive attributes needed to students when solving mathematics and science multiple-choice items for the eighth grader of a middle school. Based on the Cognitive diagnosis theory, the cognitive attributes are extracted and the mastering of cognitive skills would be analyzed. Second, this study analyze the relationship between the cognitive attributes of mathematics and science. Third, we will attempt to identify which attributes influence the achievements of both subjects. This will determine which ability in mathematics can help students to study science and, conversely, which scientific ability can help to enhance the study of mathematics. In doing so, it can help introduce integrated teaching of mathematics and science.

Ⅱ. Cognitive attributes in mathematics and science

In order to explore the cognitive attributes we referenced Trends in International Mathematics and Science Study (TIMSS) framework. TIMSS assesses achievement levels in mathematics and science at both the fourth and eighth grade and the mathematics and science assessment frameworks for TIMSS 2007 are organized around two dimensions: content domains and cognitive domains. The content domains specify the domains or subject matter to be assessed within the framework of mathematics and science. The cognitive domains describe the sets of behaviors that are expected of students as they engage in mathematics or science content.

The need to describe the ability of students to solve problem plays a crucial role in the development of an assessment like TIMSS 2007, since these behaviors are vital in ensuring that the survey covers the appropriate range of cognitive skills across the cognitive domains. The first category of content domains is knowing. It covers the facts, procedures, and concepts students need to know. The second category, applying, focuses on the ability of students to employ knowledge and conceptual understanding in order to solve problems or answer questions. The third category, reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems (IEA, 2005; Mullis et al., 2005). Each category, in turn, consists of sub-level factors, as shown in <Table 1>, which can be identified and tested. Our study will refer to these sub-level factors to identify and analyze attributes that are commonly tested by the questions on our tests.

The realm of mathematical and scientific cognition is categorized as three categories with the same title. However, the definitions on mathematics and science are different. The abilities required to solve the problems are also different. See <Table 1>.

〈Table 1〉 TIMSS 2007 cognitive domain framework

Cognitive Domain	Attribute in Mathematics	Attribute in Science
Knowing	Recall Recognize Compute Retrieve Measure Classify/Order	Recall/Recognize Define Describe Illustrate with Examples Use Tools and Procedures

Cognitive Domain	Attribute in Mathematics	Attribute in Science
Applying	Select Represent Model Implement Solve Routine Problems	Compare/Contrast/Classify Use Models Relate Interpret Information Find Solutions Explain
Reasoning	Analyze Generalize Synthesize/Integrate Justify Solve Non-Routine Problems	Analyze/Solve Problems Integrate/Synthesize Hypothesize/Predict Design/Plan Draw Conclusions Generalize Evaluate Justify

What is assessed in mathematics and science, and how they are assessed provides the clearest indication of what is valued about formal education (Hamilton, 2003). The advantage of using multiple-choice problems is that they can evaluate a student's ability involving many problems within a short period of time, but they are criticized due to the emphasis on evaluating facts and knowledge. However, the various abilities of each student can be checked through the multiple-choice problems if an analysis on the evaluation results is applied properly. Whenever appropriate, this study would propose to utilize the cognitive diagnosis theory.

Ⅲ. Cognitive diagnosis theory

Cognitive diagnosis theory was developed to evaluate examinees with respect to their levels of competence in each attribute, such as knowledge or skills. The purpose of this theory is to provide individual feedback to students, teachers, or parents regarding each student's mastery of the attributes measured by the assessment. Therefore, this theory can be described as diagnostic theory regarding students performance. Using this theory, students' mastery of each attribute can be diagnosed and their learning progress can be estimated (Embretson, 1990; DiBello, Stout, & Rousses, 1995;

Tatsuoka, 1995).

If the various skills or attributes can be evaluated through the test, parents and teachers can understand the status of students' abilities or attributes. This will make the test a very useful tool for educational communication. In previous studies, the results of the test have been calculated to provide a single score; however, if the cognitive diagnosis theory is used, the students' individual results of the mastery of various attributes, along with the overall results, can be used to evaluate the students qualitatively. If the mathematics and science cognitive abilities can be diagnosed, we can determine which abilities are necessary to learn each subject.

Many of cognitive diagnostic models were developed to diagnosis students' ability such as Rule Space Model, Unified Model, DINA model, Fusion model and so on under the Cognitive diagnosis theory (Tatsuoka, 1990; Templin & Henson, 2006; von Davier, 2005; Rupp, Templin, & Henson, 2010). Cognitive diagnosis theory has been utilized in many recent studies by applying cognitive diagnostics models. For example, the Cognitive Diagnostic model has been employed to analyze students' ability using TIMSS data (Tatsuoka, Corter, & Tatsuoka, 2004). Dogan and Tatsuoka (2008) reported a comparison of the mastery of the mathematics attributes between students in Turkey and the United States using TIMSS-R results using Rule Space Model. Kim, Kim, and Song (2008) analyzed the mastery of attributes for mathematics among ninth grade Korean students using the Fusion model. Instead of giving students a total score that represents an overall ability, specific information regarding the mastery of each attribute made it more effective to plan their learning.

For this study, science and mathematics cognitive abilities will be analyzed using the Fusion model. The Fusion model was used for the analysis due to the easy access to the computer program as well as the availability of the number of attributes that were analyzed. A series of processes are pertinent for utilizing the Fusion model. First, proper assessment items that can measure students' science and mathematics cognitive abilities should be developed. Second, each attribute measured by each item needs to be set. The attributes that measure students' mastery are related to several cognitive behaviors; therefore, it needs to be detailed and concrete. This step is for subject specialists who develop and analyze the items and teachers who evaluate students. Third, once the attributes are set, a Q-matrix should be created. Q-matrix is a matrix showing the relationship between attributes that items are purported to measure and the items themselves (Tatsuoka, 1983, 1990, 1995). If the attribute is needed to solve the item, the element of Q-matrix is 1; if not, it is 0. The row represents the number of attributes and the column represents the number of items. Fourth, the item parameters regarding each attribute and students' parameter of mastery of each attribute should be estimated.

The Fusion Model, so called Reparameterized Unified Model, is one of the Cognitive diagnostic models based on item response theory defining the probability of observing the response of examinee j to item i, given the examinee's ability parameters and item parameters. The equation of the Fusion model is as follows:

$$P(X_{ij}=1|\alpha_j,\theta_j)=\pi_i^*\prod_{k=1}^{K}r_{ik}^{*(1-\alpha_{ik})\times q_{ik}}P_{C_i}(\theta_j)$$

In this equation, q_{ik} is the attribute k which is measured by item i. If X_{ij}=1, it indicates that examinee j has mastered attribute k; otherwise, q_{ik}= 0. The symbol indicates the response of examinee j to item i, where x = 1 indicates a correct response and x = 0 indicates an incorrect response. The parameter π_i^* is the probability of correctly applying all items i with the required attributes. It can be explained as the probability of an examinee, who has mastered all attributes for item i, to correctly apply those attributes when solving item i. This application is interpreted as the Q-based item i difficulty (Hartz, Roussos, & Stout, 2002). Based on the Fusion model, students' performance can be diagnosed by examining whether or not the individual student have mastered the attributes. The mastery of each attributes can be calculated by probability from the Fusion model.

Using the Fusion model, the each item can be analyzed as well because the Fusion model estimated two item parameters such as r_{ik}^* and π_i^*. The parameter r_{ik}^* is the proportional parameter representing the ratio of the likelihood of a correct answer, given mastery versus non-mastery (Hartz et al., 2002). It can be interpreted as the item i discrimination parameter for attribute k (Hartz, 2002; Hartz et al., 2002), $P_{C_i}(\theta_j)$ is the probability of applying the skills correctly, particularly the skills not specified by the Q-matrix. Using these item parameters, each of the item can be examined whether it contains the right attributes and how well it estimates those attributes.

Ⅳ. Method and procedure

1. Test development and administration

The test used in this study was developed by three mathematics teachers and three science teachers, each of whom had more than five years of teaching experience in middle school. The test was complied through joint discussions between the researchers and the teachers. Each problem consists of multiple-choice responses. The tests were designed to be completed in 45 minutes, which is the length of a class time in a typical Korean middle school. The contents for mathematics included “properties of triangle”, “properties of square”, and the “similarity of figure”. The science contents included “structure and function of plant”, “stimulus and response”, the “history of earth and the change of the earth’s crust”, “electricity” and the “separation of mixture”.

Test participants were eighth graders at a middle school in Seoul. This study analyzed students’ answers of 17 mathematical multiple-choice questions in geometry and 31 multiple-choice questions in science. While the actual tests combine both multiple-choice and constructed-response items, this study focused on only the multiple-choice items since the level of cognitive skills and abilities each question requires could be differentiated (Thissen, Wainer, & Wang, 1993). Of the 466 students recruited from 14 classes, 239 were male and 227 were female.

2. Identifying attributes of items

Each test item contains attributes necessary to solve it. The types of behaviors necessary to solve mathematical and scientific problems are also meaningful, and were identified as attributes to be analyzed. To this end, this study selected several sub-level factors taken from <Table 1> (Mullis et al., 2005) as the attributes to be analyzed. The TIMSS 2007 framework contains specific factors of cognitive skills and abilities, enabling the attributes to be identified.

Of the sub-level factors in the cognitive domains shown in <Table 1>, this study chose “Recognize”, “Analyze”, “Justify”, “Synthesize/Integrate”, and “Recall” as the attributes tested by the questions included in the mathematics test items. Each attribute is described within the TIMSS framework as follows:

Recognize(M): Recognize mathematical objects, shapes, numbers and expressions. Recognize mathematical entities that are mathematically equivalent (e.g., equivalent familiar fractions, decimals and percents; and different orientations of simple geometric figures).

Analyze(M): Determine and describe or use relationships between variables or objects in mathematical situations; decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; visualize transformations of three-dimensional figures; and make valid inferences from given information.

Synthesize/Integrate(M): Combine (various) mathematical procedures to establish results, and combine results to produce a further result. Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas.

Justify(M): Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.

Recall(M): Recall definitions; terminologies; number properties; geometric properties; and notations (e.g., $a \times b = ab$, $a + a + a = 3a$).

For the science portion, this study has chosen “Recall/Recognize”, “Interpret Information”, “Find solution”, “Explain”, and “Integrate/Synthesize” as the attributes. Each science attribute is described within the TIMSS framework as follows:

Recall/Recognize(S): Make or identify accurate statements about science facts, relationships, processes, and concepts; identify the characteristics or properties of specific organisms, materials, and processes.

Interpret Information(S): Interpret relevant textual, tabular, or graphical information in light of a science concept or principle.

Find Solution(S): Identify or use a science relationship, equation, or formula to find a qualitative or quantitative solution involving the direct application/demonstration of a concept.

Explain(S): Provide or identify an explanation for an observation or natural phenomenon, demonstrating understanding of the underlying science concept, principle, law, or theory.

Integrate/Synthesize(S): Provide solutions to problems that require consideration of a number of different factors or related concepts; make associations or connections between concepts in different areas of science; demonstrate understanding of unified concepts and themes across the domains of science; integrate mathematical concepts or procedures in the solutions to science problems.

3. Constructing Q-matrix

In order to use the Fusion model, it is necessary to create a Q-matrix to connect each question item to the attributes to be explained. To create the Q-matrix about mathematics items, we first ascribed attributes to the questions. The questions initially had the “Solve Routine Problems(M)” attribute, but not the “Justify(M)” attribute that was presented in <Table 2>. The Fusion model, however, showed that these questions did not adequately reflect the chosen attributes. After consulting the mathematics teachers, we excluded the “Solve Routine Problems(M)” attribute from the questions, adding the “Justify(M)” attribute instead and modifying the connections between the questions and the corresponding attributes.

The example of attributes connected to the test items is as follows (Kim, 2009). Math item of 〔Figure 1〕, for example, is a question that requires the attributes of “Recall” and “Justify”. Students are asked to justify which of the following statements, when reversed, is true. In doing so, they must recall the definition and properties of each object.

What is the statement that its reverse is the truth?
① A regular triangle is an equilateral triangle.
② The divisor of 12 is the divisor of 4.
③ If $x = 3$, then $2x - 6 > 0$.
④ If $a > b$, then $ac > bc$.
⑤ If $a = b$, then $ac = bc$.

〔Figure 1〕 Math item

<Table 2> shows the Q-matrix created to connect the seventeen questions to the five cognitive attributes.

〈Table 2〉 Q-matrix of mathematics items

Item	Recall (M)	Recognize (M)	Analyze (M)	Justify (M)	Synthesize/ Integrate(M)	# of attributes
1			1		1	2
2			1	1		2
3	1	1	1			3
4		1	1			2
5		1		1		2
6	1	1		1		3
7		1		1	1	3
8				1	1	2
9	1	1	1			3
10		1			1	2
11		1	1			2
12	1	1	1			3
13		1		1	1	3

Item	Recall (M)	Recognize (M)	Analyze (M)	Justify (M)	Synthesize/ Integrate(M)	# of attributes
14		1	1			2
15		1	1			2
16		1	1			2
17		1		1		2
Total	4	14	10	7	5	

Because Q-matrix is a matrix showing the relationship between attributes that items are purported to measure and the items themselves (Tatsuoka, 1983, 1990, 1995), <Table 2> shows 17 items were matched to each of these five cognitive attributes. In order to solve each question, students used two or three cognitive attributes.

Attributes were also ascribed to the questions to create the science items for the Q-matrix. In addition to the five attributes listed in <Table 3>, the "Draw conclusion" attribute was added as well. The inappropriateness of the conversion of parameters for the six attributes was revealed through the Fusion model analysis. After consultation, the science teachers decided to use five of the six attributes by excluding the "Draw conclusion(S)" attribute. The connections of problem items with the attributes were also modified.

Here is an example of connecting attributes to the test items. Science item 2 in 〔Figure 2〕 calls for the attributes of "Explain(S)" and "Integrate/Synthesize(S)". The attributes of "Integrate/Synthesize(S)" are required because the different concepts of the computer and the human body need to be understood through connection. The attributes of "Explain(S)" are required because the reasoning behind the connection of the two concepts needs to be explained.

Which of the following selections compares the parts of a computer correctly with the parts of a human body?

	Computer	Body of person
A	An input device	Brain
B	An output device	A locomotive organ
C	A mainframe computer	A sensory organ
D	Connecting cable	Nerves

① A, B ② A, B, C ③ B, C
④ B, D ⑤ A, B, C, D

〔Figure 2〕 Science item 2

<Table 3> shows the Q-matrix created to connect the thirty-one questions to the five cognitive attributes.

〈Table 3〉 Q-matrix of science items

Item	Recall/ Recognize(S)	Interpret Information(S)	Find Solution(S)	Explain(S)	Integrate/ Synthesize(S)	# of attributes
1			1	1		2
2			1		1	2
3			1		1	2
4				1	1	2
5		1	1			2
6	1			1		2
7	1					1
8				1		1
9	1					1
10	1					1
11	1	1			1	3
12				1	1	2
13		1				1
14	1	1				2
15	1				1	2
16	1				1	2
17	1					1
18	1					1
19				1		1
20				1		1
21	1	1				2
22	1			1		2
23	1					1
24					1	1
25				1		1
26	1					1

Item	Recall/ Recognize(S)	Interpret Information(S)	Find Solution(S)	Explain(S)	Integrate/ Synthesize(S)	# of attributes
27			1			1
28		1				1
29			1		1	2
30		1				1
31				1		1
Total	14	7	6	10	9	46

In <Table 3>, it shows that 31 items were matched to each of five cognitive attributes. In order to solve each question, one or more cognitive attributes were needed. For example, to solve item 11, students need to use three cognitive attributes such as Recall/Recognize(S), Interpret Information(S), and Integrate/Synthesize(S). In solving science items, students most frequently need to use Recall/Recognize(S).

4. Analysis method

The item parameters and person parameters of the Fusion model were analyzed by using the Markov Chain Monte Carlo (MCMC) estimation method of the Arpeggio program (Hartz et al., 2002). Based on the Fusion model, the parameter of each question with their corresponding attributes were estimated. The investigators used the autocorrelation of the estimated parameters in the MCMC estimation to decide whether or not a test item should be included. If the item parameter of each question chosen for this study and the student parameter of each participant was 0.2 or less, the questions were included as test items. The relatively high values of the estimated item parameters (ranging between 0.79 and 0.99) indicated that each question strongly called for the mathematical attributes indicated by the Q-matrix. Most of the estimated student parameters ranged from 0.25 to 0.90, showing clearly that each question was linked to each attribute. In other words, estimating the parameter of each question showed that the questions included in this study differentiated each of the attributes.

A frequency analysis was conducted to determine what attributes students have mastered. The inter-attribute correlations were determined by applying Pearson’s correlation coefficient to ppm, representing the probability each student had in mastering each given attribute for mathematics and science.

To determine the relationship between each attribute and the students' performance, each attribute was treated as an independent variable and the overall score as a dependent variable. A regression analysis involving these variables was then calculated for mathematics and science.

Ⅴ. Results

1. Cognitive attributes in mathematics and science

1) Mathematics

The average score for the 17 questions was 12.64, with a standard deviation of 3.75. <Table 4> shows the mathematics cognitive skills mastered by students, with their corresponding data and ratios of students. In this study, 69.5% of the students mastered "Recall(M)"; 77.0%, "Recognize(M)"; 54.1%, "Analyze(M)"; 41.6%, "Justify(M)"; and 76.8%, "Synthesize/Integrate(M)." When presented in descending order to determine how well each attribute was mastered by the students, they were as follows: "Recognize(M)", "Synthesize/Integrate(M)", "Recall(M)", "Analyze(M)", and "Justify(M)".

〈Table 4〉 Percentage of students who mastered the cognitive attributes in math (Kim, 2009)

Attribute in Math	Mastering (%)	Not Mastering (%)
Recall(M)	324 (69.5)	142 (30.5)
Recognize(M)	359 (77.0)	107 (23.0)
Analyze(M)	252 (54.1)	214 (45.9)
Justify(M)	194 (41.6)	272 (58.4)
Synthesize/Integrate(M)	358 (76.8)	108 (23.2)

<Table 5> shows the correlations among the cognitive attributes required for mathematics. All five cognitive attributes were correlated with one another, within the significance level of .01.

〈Table 5〉 Correlation coefficients between mathematics cognitive attributes (Kim, 2009)

	Recall(M)	Recognize(M)	Analyze(M)	Justify(M)
Recognize(M)	.793**	1		
Analyze(M)	.757**	.701**	1	
Justify(M)	.710**	.603**	.796**	1
Synthesize/Integrate(M)	.731**	.792**	.684**	.600**

* p < .05, **p < .01, ***p < .001

Two attributes with the strongest correlations were “Justify(M)” and “Analyze(M)” (.796). The ability to discern whether a given statement is true or false was closely related to the ability to find answers to non-routine problems. In other words, because both attributes belonged to the “reasoning” area within the TIMSS 2007 framework, the correlation was expected to be high. Two attributes with the lowest correlations were “Justify(M)” and “Synthesize/Integrate(M)” (.600). The ability to discern whether a statement is true or not was high not related to the ability to synthesize and integrate multiple pieces of information.

2) Science

<Table 6> shows the levels of science cognitive skills mastered by students.

〈Table 6〉 Percentage of students with mastered cognitive attributes in science

Attribute	Mastering (%)		Not Mastering (%)	
Recall/Recognize(S)	282	(60.5)	184	(39.5)
Interpreting Information(S)	304	(65.2)	162	(34.8)
Find Solution(S)	217	(46.6)	249	(53.4)
Explain(S)	275	(59.0)	191	(41.0)
Integrate/Synthesize(S)	218	(46.8)	248	(53.2)

The range of mastering attributes was between 46.6% and 65.2%: 60.5% for “Recall/Recognize(S)”; 65.2% for “Interpreting Information(S)”; 46.6% for “Find Solution(S)”; 59.0% for “Explain(S)”; and 46.8% for “Integrate/Synthesize(S)”. Ranking in descending order, the results showed how well each attribute was mastered by the students: “Interpreting Information(S)”, “Recall/Recognize(S)”, “Explain(S)”, “Integrate/Synthesize(S)”, and “Find Solution(S)”.

"Interpreting Information(S)" was the easiest attribute to master and "Find Solution(S)" was the most difficult.

<Table 7> shows the correlations among the cognitive attributes required for solving scientific problems.

〈Table 7〉 Correlation coefficients between science cognitive attributes

	Recall/ Recognize(S)	Interpret Information(S)	Find Solution(S)	Explain(S)
Interpreting Information(S)	.811**	1		
Find Solution(S)	.738**	.751**	1	
Explain(S)	.824**	.817**	.798**	1*
Integrate/Synthesize(S)	.746**	.741**	.809**	.788**

*p < .05, **p < .01, ***p < .001

All of the five cognitive attributes were correlated with one another, within the significance level of .01. The range of correlation coefficients was between .738 and .824. The two attributes with the strongest correlations were "Recall/Recognize(S)" and "Explain(S)" (.824). The ability to recall information from past memories was relative to providing an explanation for an observation or natural phenomenon. The two attributes with the lowest correlations were "Recall/Recognize(S)" and "Find Solution(S)" (.738).

2. Correlation of attributes between mathematics and science

The five attributes in each of mathematics and science were correlated with one another within the significance level of 0.01 as <Table 8>.

〈Table 8〉 Correlation coefficients between science attributes and mathematics attributes

Science / Math	Recall/ Recognize(S)	Interpreting Information(S)	Find Solution(S)	Explain(S)	Integrate/ Synthesize(S)
Recall(M)	.556**	.560**	.534**	.552**	.539**
Recognize(M)	.580**	.585**	.561**	.576**	.554**
Analyze(M)	.594**	.562**	.592**	.586**	.572**
Justify(M)	.540**	.519**	.622**	.565**	.555**
Synthesize/ Integrate(M)	.577**	.594**	.534**	.545**	.532**

** p < .01

The correlation coefficients were within the range of .519 – .622, showing moderate relationships. The attributes with the highest correlation coefficients were “Find Solution(S)” and “Justify(M)” (.622). The attributes with the lowest correlation coefficients were “Interpreting Information(S)” and “Justify(M)” (.519). Findings of previous studies showed countries with high mathematics achievement was high in science achievement at TIMSS or PISA of international comparison evaluation (OECD, 2007; Mullis et al., 2008). High correlation of mathematics and science cognitive attributes is consistent with correlation of achievement scores of both subjects

3. Effect of attributes on achievement of mathematics and science

In this study, we determined the relationship between the cognitive attributes of mathematics and science and the achievement levels of these two subjects. This was determined by investigating how each mathematics and science attribute composition could predict overall mathematical achievement. A regression analysis was analyzed to ascertain how the values on the independent variable could help predict the changes in the values on the dependent variable. In this case, the dependent variable was the sum of the scores (representing mathematical achievement), and the independent variable was the probability of mastering (ppm) 10 cognitive attributes. Using stepwise regression, six models were created. Of these, the fifth model had the strongest explicatory power and contained cognitive attributes containing five mathematics attributes. For this model, there was 95.9% chance of explaining the changes in mathematical performance with the coefficient of determination. See <Table 9> and <Table 10>.

〈Table 9〉 Mathematics achievement: ANOVA

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	6266.240	5	1253.248	2141.561	.000
Residual	269.193	460	.585		
Total	6535.433	465			

〈Table 10〉 Regression results for mathematics achievement

Model	B	Standard Error	β	T	p	R^2
(constants)	5.464	.103		53.241	.000	.959
Recognize(M)	4.376	.187	.427	23.408	.000	

Model	B	Standard Error	β	T	p	R^2
Analyze(M)	2.609	.173	.276	15.118	.000	
Justify(M)	2.430	.158	.250	15.338	.000	
Synthesize/Integrate(M)	1.315	.183	.118	7.195	.000	
Recall(M)	.450	.227	.036	1.981	.048	

The order in the standardized regression coefficient was “Recognize(M)”, “Analyze(M)”, “Justify(M)”, “Synthesize/Integrate(M)”, and “Recall(M)”. A higher standardized coefficient of regression for any given attribute represented a larger attribute effect on mathematical achievement.

Regression analysis was also performed to determine how much skill for each attribute can be expected for achievement in science. The stepwise method was applied, using science scores as dependent variables and the five attributes each in science and mathematics as the independent variables. The five attributes in science and the attributes of “Analyze(M)” and “Recognize(M)” in mathematics were inputted. The remaining three attributes in mathematics were removed. The quantity of change in the score of science was explained by 92.3% of the decision coefficient. See <Table 11> and <Table 12>.

〈Table 11〉 Science achievement: ANOVA

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	21747.611	7	3106.802	786.291	.000
Residual	1809.653	458	3.951		
Total	23557.264	465			

〈Table 12〉 Regression results for science achievement

Model	B	Standard Error	β	t	p	R^2
(constants)	10.887	.219		49.805	.000	.923
Explain(S)	2.901	.450	.183	6.444	.000	
Interpreting Information(S)	4.599	.446	.265	10.302	.000	
Integrate/Synthesize(S)	2.808	.409	.170	6.870	.000	
Recall/Recognize(S)	3.052	.403	.197	7.569	.000	

Model	B	Standard Error	β	t	p	R^2
Find Solution(S)	2.566	.432	.150	5.937	.000	
Analyze(M)	1.130	.352	.063	3.213	.001	
Recognize(M)	1.195	.376	.061	3.181	.017	

The regression coefficient in “Explain(S)” was 2.901, showing significant influence on the science score within .01 significance level. The standardized regression coefficient has a bigger influence on the science score. For this study, the order of attributes was “Explain(S)”, “Interpret Information(S)”, “Integrate/Synthesize(S)”, “Recall/Recognize(S)”, “Find Solution(S)”, “Analyze(M)”, and “Recognize(M)”. The five attributes of science produced a strong influence on the science score, but it is notable that the mathematics attributes of “Analyze(M)” and “Recognize(M)” were helpful in predicting the science score as well.

Ⅵ. Conclusion and Discussion

This study examined the cognitive abilities that students utilize when solve mathematics and science problems. The cognitive abilities included in solving mathematics and science problems were identified through the Cognitive diagnosis theory. There are five attributes in the contents of geometry in mathematics, and students mastered each attribute in the order of “Recognize(M)”, “Synthesize/Integrate(M)”, “Recall(M)”, “Analyze(M)”, and “Justify(M)”. Five attributes were also measured in solving science problems. Students mastered each attribute in the order of “Interpreting Information(S)”, “Recall/Recognize(S)”, “Explain(S)”, “Integrate/Synthesize(S)”, and “Find Solution(S)”. It implies that cognitive attributes may be easily taught to students following this order in mathematics and science classes. Namely, teacher first may teach contents with easy cognitive attributes and later may teach contents with difficult cognitive attributes. Teaching of these order is thought to be useful method that student understand easily content.

There was a high correlation among the cognitive attributes in both mathematics and science. The highly correlated attributes in mathematics and science were “Analyze(M)” and “Find Solution(S)”, “Analyze(M)” and “Recall/Recognize(S)”, “Analyze(M)” and “Explain(S)”. Namely, Analyze(M) in mathematics highly related to solving the science items. “Analyze(M)”, which requires an ability to break down a diagram into meaningful data and infer from the given information to solve a problem, is related to finding a solution of problem-solving based on the application of scientific concepts.

Thus, the ability to solve problems of mathematics or science that was found in cognitive attributes was utilized commonly in both subjects. Several of these attributes are closely related to each other.

This analysis showed that the five cognitive attributes in mathematics were related to mathematical achievement. In addition to the five attributes in science, “Recognize(M)” and “Analyze(M)” attributes were the other variables influential in science achievement. To solve science problems, several abilities are necessary: the ability to recognize objects, shapes, numbers, and expressions; to check entities that are mathematically equivalent; to determine and describe or use relationships between variables or objects; and to make valid inferences from given information. Based on this inference, we presume that the abilities of recognizing and reasoning to be used in mathematics are also necessary in the study of science.

By using the Cognitive diagnostic models to analyze the students' performance and proficiency, more accurate information can be provided to the individual students as well as more actionable feedback can help students to learn more efficiently. Song et al. (2011) used Korea National Assessment of Educational Achievement data to analyzed students proficiency and gave some implication of developing the student profile reports. Like this study, diagnosing students performance can be used very efficiently by giving individual students profile of attributes. This can be useful for developing individualized teaching and learning methods for each student.

Recent research has tried to compose classes based on an integrated relationship of mathematics and science (Wilhelm & Walters, 2006). This study attempts to provide such an integrated education. House (1997) describes a continuum extending from pure mathematics on the one end to pure science on the other, that is, pure mathematics, science as part of math, math and science integrated, math as part of science, and pure science. In the middle, we find an integrated classroom where it would be difficult to distinguish whether it is a “mathematics” or a “science” class. House describes the center of this continuum as “Sciematics”, where both science and mathematics are equally respected, with neither discipline treated as either superior or inferior to the other. The principles and content of both subjects remain the same: science continues to investigate the natural world, while mathematics continues to explore numbers, quantities, shapes, data, space, patterns, and structure. Science and mathematics are taught and studied mutually and supportively. Even without creating a new subject such as “Sciematics,” researchers should consider a mutual relationship between mathematics and science involving similar cognitive abilities.

The Ministry of Education, Science and Technology (MEST) and the Korea Foundation for the

Advancement of Science and Creativity (KOFAC) recruited teachers' research councils for STEAM (science, technology, engineering, art, and mathematics) education in order to introduce STEAM education in elementary, middle, and high schools from 2011. This policy shows that MEST encourages the consilience of several subjects. But to achieve this, the interdisciplinary characteristics should be grasped by teachers with teaching and learning methods, or education contents. This study showed high correlation of mathematics and science cognitive attributes. Therefore teaching strategies using mathematics and science cognitive attributes with high correlation is considered to be effective in teaching students.

Though mathematics and science teachers may teach their own specialized subject, they should recognize that students experience a similarity of abilities when studying mathematics and science even though the contents are different. Mathematics has historically been developed using the material of science, and science with the utilization of mathematics as knowledge and technology. The procedures that students experience when they study mathematics and science are not different either. Consequently, the teacher needs to teach each subject by utilizing the materials to be found from each discipline. The teacher also needs to guide students to learn concepts from the other subject while teaching in his or her area of expertise. In addition, each teacher must try to improve the students' abilities to solve problems that involve common cognitive abilities. For these reasons, an integrated class of mathematics and science for contents that are not specialized would be a good alternative form of teaching.

Students should understand and utilize the principles of both math and science observed in everyday life; those without any experience in problem-solving will have difficulties. We need to develop and provide teachers with an integrated mathematics and science education. Continued emphasis on improving content knowledge is also a prerequisite to enable teachers to integrate content in both mathematics and science. In addition, future research needs to develop instructional strategies to enable an integration of both mathematics and science contents.

Result of high correlation at mathematics and science cognitive attributes may give research theme about gender difference of both subjects. Generally, compared to other subjects, gender difference exists in mathematics and science subjects. Research that mastering difference of any cognitive attributes in both subjects create the gender difference will give useful information.

Finally, it was strongly believed that the findings from this study will provide useful information in creating an integrated program to identify the mathematical and scientific cognitive abilities for solving mathematical problems, and conversely, to identity the scientific and mathematical cognitive abilities to solve scientific problems.

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요약

수학과 과학에서의 인지 능력 간의 관계

김 선 희(신라대학교 조교수)
김 수 진(한국교육과정평가원 연구위원)
김 지 영(명일중학교 교사)

본 연구에서는 수학과 과학 두 교과에서 평가되고 있는 인지 능력 간에 어떤한 관련성이 있는지 인지 진단 이론인 퓨전 모델에 기초하여 확인하였다. 인지 진단 이론은 검사에 의해 측정되는 특정 속성을 학습자가 어느 정도 숙달했는지 알려줄 수 있는 이론이다. 본 연구는 중학교 2학년 정규고사에서 수학과 과학의 선다형 문항을 분석하여 인지 속성을 각각 추출한 후 인지 속성의 숙달 여부와 수학과 과학의 인지적 속성 간의 상관을 분석하였고 회귀 분석을 통해 수학과 과학의 인지적 속성이 각 과목의 성취에 미치는 영향을 분석하였다. 분석 결과 수학과 과학의 인지적 속성 간에 높은 상관이 나타났으며 몇 가지 수학적 속성은 과학적 문제를 해결하는 데 관련이 있는 것으로 나타났다. 이를 볼 때, 수학 교과에서 신장되어야 할 몇 가지 인지적 능력이 과학 학습에서도 필요한 것임을 알 수 있다. 최근 수학과 과학 교과의 관련성에 기초하여 수업을 통합적인 관점에서 구성하여 진행하려는 노력이 이루어지고 있는데 본 연구는 이러한 융합 교육에 있어서 시사점을 제공해 준다고 할 수 있다.

주제어 : 수학, 과학, 인지 능력, 인지 진단 이론, 성취도